A Six-Port Reflectometer and its Complete Characterization by Convenient Calibration Procedures

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Abstract—A simple six-port reflectometer and two methods of calibration using either $5\frac{1}{2}$ or $6\frac{1}{2}$ standard terminations are described. High precision is obtained using diode detectors, which are calibrated *in situ* against a single thermistor power meter. The addition of a few extra components provides a moderately accurate visual display in real time over the 1–8-GHz band.

I. Introduction

During the past decade, microwave impedance instrumentation has seen the emergence of six-ports as an alternative to conventional network analyzers. Whereas conventional analyzers have a degree of electronic complexity commensurate with the need to measure phase to high accuracy, six-ports are inherently simple and stable devices which do not need to be constructed of precision components, and require only power meters in order to measure complex impedances. The theory of six-ports is complete so that, at least in principle, they are capable of returning error free results after a suitable calibration procedure has been implemented.

The desirable features sought in the design of a six-port reflectometer include high accuracy, low cost, ease of calibration, and if possible, a real-time visual display of the reflection coefficient being measured. These features are not necessarily compatible, as can be deduced from the many papers in the literature dealing with six-ports and their calibration [1]–[9].

If thermistor power meters are used to achieve the high linearity, low noise, and low drift needed for high accuracy, the RF drive to the six-port needs to be high level, which increases the system cost. The configuration of a six-port reflectometer designed to provide a real-time visual display of reflection coefficient by analog means [10] is not necessarily suited to the application of six-port theory involving numerical computation [5].

Although a visual display is not necessary for six-port measurements, its provision does increase the utility of the six-port so that it may be used for rapid, though less accurate, measurements during development work. Such a display is also useful to detect instabilities of any kind (such as loose connectors), and to make relative measurements (comparisons) in real time.

The purpose of this paper is to describe two calibration procedures and their application to a relatively inexpensive six-port capable of measuring reflection coefficients with high accuracy. Unlike some calibration procedures [4], [9], the calibration constants are obtained from matrix equations derived directly from the general six-port equation. Diode detectors calibrated *in situ* are used at comparatively low levels as power meters.

II. Basic Concepts

The general six-port in Fig. 1 is described by an equation developed by Hoer [5] and confirmed by Woods [7] as[1]

$$\Gamma = r + jx = \sum_{i=1}^{4} (F_i + jG_i)P_i \Big/ \sum_{i=1}^{4} H_i P_i \tag{1}$$

where Γ is the measurand, P_i is the reading of the ith power meter, and F_i, G_i, H_i are real constants. Without loss of generality, these constants may be normalized by setting $H_4 = 1$, leaving eleven real constants to be determined from (1) by a suitable calibration procedure involving the successive connection of known terminations (standard reflections) to the measuring port.

By separating the real and imaginary parts, (1) may be rewritten as the two equations

$$\sum_{i=1}^{4} F_i P_i - r \sum_{i=1}^{3} H_i P_i = rP_4 \tag{2a}$$

$$\sum_{i=1}^{4} G_i P_i - x \sum_{i=1}^{3} H_i P_i = xP_4. \tag{2b}$$

It is these two equations which are used in the six-port calibration.

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[1]This equation applies to linear six-ports but does not require the six-port to be reciprocal, or even realizable. The effects of reciprocity and realizability on the number of degrees of freedom of the six-port equation does not appear to have been investigated.

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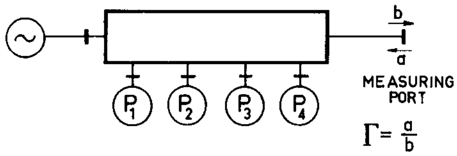

Fig. 1. Schematic of an arbitrary linear microwave network having an input port, a measuring port, and four additional ports to which power indicators (P_1 to P_4) are connected.

III. Two Calibration Methods

The calibration method of Woods [7] uses seven (complex) standards to obtain the eleven (real) six-port coefficients. Using his approach, the practical realities of electronic noise and imperfections in the standards will lead to numerical contradictions.

We propose two calibration methods which use as calibrating standards four and five phased (offset) short-circuits, a matched load, and an intermediate termination with a reflection coefficient $0.3 \leqslant |\Gamma| \leqslant 0.7$, as suggested by Woods [7]. However, in effect, only $5\frac{1}{2}$ (complex) standards are used to determine the eleven (real) coefficients exactly, or $6\frac{1}{2}$ (complex) standards with provision to accommodate imperfections.

The phased short-circuits may be regarded as absolute standards, since their behavior is calculable from transmission line dimensions, with small allowances for line losses (see Appendix A). The matched load is an absolute standard whose zero-magnitude reflection coefficient may be realized as the centroid of the locus of the reflection coefficient of an imperfect match as its phase is varied.[2] The magnitude of an intermediate reflection coefficient is calculable for a reduced height waveguide incorporating a sliding load, or for a sliding load in a coaxial line with stepped inner conductor. Alternatively, the magnitude of this intermediate reflection standard may be measured with great accuracy using a tuned reflectometer [11]. However, the calculation or accurate measurement of the phase of an intermediate reflection coefficient is more difficult, particularly as it must be referred to an arbitrarily nominated phase plane for the six-port calibration.

Both of the following calibration methods make use of inherent redundancies to avoid the need to assign an accurate value to the phase of the intermediate calibrating reflection coefficient. It should be emphasized, however, that it is not necessary to make use of these redundancies if the magnitudes and phases of all the calibrating terminations are known. In the following sections, Γ_n will be used to denote the reflection coefficient of the nth calibrating termination, where

$$\Gamma_n = r_n + jx_n = |\Gamma_n| \exp(j\theta_n) \tag{3}$$

and P_{ni} will be used to denote the reading of the ith power meter when the nth calibrating termination is connected to the measuring port of the six-port.

A. Method A

If six standard terminations are connected successively to the measuring port, twelve equations in the eleven unknown six-port constants are obtained from (2a) and (2b). These constants may be determined from eleven of the equations, and a measure of the calibration uncertainty obtained by examining the extent to which the redundant twelfth equation is satisfied.

If only five standard terminations are known, and either the real or imaginary part of the reflection coefficient of a sixth, eleven equations in the eleven six-port unknowns are obtained, again from (2a) and (2b). These may be solved for the six-port constants, and a twelfth equation is then available from either (2a) or (2b) to provide a full description of the sixth termination. The number of calibrating standards used may be said to be $5\frac{1}{2}$.

Instead of providing either the real or imaginary part of the reflection coefficient of the sixth standard, we elect to provide only its magnitude. Because of the form of (2a) and (2b), it then becomes necessary to use all twelve equations to determine the six-port constants, and incidentally, the phase of the sixth standard reflection. Four phased short-circuits ($\Gamma_1, \cdots, \Gamma_4$), a matched load ($\Gamma_5$), and an intermediate termination ($0.3 \leqslant |\Gamma_6| \leqslant 0.7$) are used as calibrating standards. An initial estimate of θ_6 is made, and a first evaluation of the six-port constants obtained from

$$\begin{bmatrix} F_1 \\ \vdots \\ F_4 \\ G_1 \\ \vdots \\ G_4 \\ H_1 \\ \vdots \\ H_3 \end{bmatrix} = \begin{bmatrix} P_{11} & \cdot\cdot & P_{14} & 0 & \cdot\cdot & 0 & -r_1P_{11} & \cdot & -r_1P_{13} \\ \vdots & & & & & & & & \\ P_{41} & \cdot\cdot & P_{44} & 0 & \cdot\cdot & 0 & -r_4P_{41} & \cdot & -r_4P_{43} \\ 0 & \cdot\cdot & 0 & P_{11} & \cdot\cdot & P_{14} & -x_1P_{11} & \cdot & -x_1P_{13} \\ \vdots & & & & & & & & \\ 0 & \cdot\cdot & 0 & P_{41} & \cdot\cdot & P_{44} & -x_4P_{41} & \cdot & -x_4P_{43} \\ P_{51} & \cdot\cdot & P_{54} & 0 & \cdot\cdot & & & \cdot & 0 \\ 0 & \cdot\cdot & 0 & P_{51} & \cdot\cdot & P_{54} & 0 & \cdot & 0 \\ P_{61} & \cdot\cdot & P_{64} & 0 & \cdot\cdot & 0 & -r_6P_{61} & \cdot & -r_6P_{63} \end{bmatrix}^{1} \cdot \begin{bmatrix} r_1P_{14} \\ \vdots \\ r_4P_{44} \\ x_1P_{14} \\ \vdots \\ x_4P_{44} \\ 0 \\ 0 \\ r_6P_{64} \end{bmatrix}. \tag{4}$$

[2] If a sliding load is moved to several equally spaced positions covering the range of $\lambda_g/2$, the averages of each power meter's readings are taken as the values that would have been obtained with a perfect match connected.

θ_6 is iterated until $|\Gamma_6|\sin\theta_6$ equals x_6 as given by the twelfth equation from (2b),

$$x_6 = \sum_{i=1}^{4} G_i P_{6i} \Big/ \sum_{i=1}^{4} H_i P_{6i}. \quad (5)$$

Two solutions differing by 180° are possible, and it is therefore necessary to have prior knowledge of the quadrant to which θ_6 belongs.

B. Method B

If seven standard terminations are connected successively to the measuring port, two matrix equations of order seven are obtained from (2a) and (2b). These equations may be solved independently for the unknown six-port constants as suggested by Woods [7]. Because his calibration method is overdetermined (eleven constants to be determined from up to fourteen equations), some of the rigidity of the system of equations has to be removed for it to work in any practical situation.

The redundancy inherent in solving fourteen linear equations in eleven unknowns is manifested in the determination of two sets $\{_aH_i\}, \{_bH_i\}$ from (2a) and (2b), respectively, each set purporting to be $\{H_i\}$. This redundancy is utilized to avoid having to accurately define the phase of the intermediate reflection coefficient. The number of standard terminations used may then be said to be $6\frac{1}{2}$.

Five phased short-circuits $(\Gamma_1, \cdots, \Gamma_5)$, a matched load (Γ_6), and an intermediate termination $(0.3 \leq |\Gamma_7| \leq 0.7)$ are used as the calibrating standards. An initial estimate of θ_7 is made, and a first evaluation of the six-port constants is obtained from

$$\begin{bmatrix} F_1 \\ \vdots \\ F_4 \\ _aH_1 \\ \cdot \\ _aH_3 \end{bmatrix} = \begin{bmatrix} P_{11} & \cdots & P_{14} & -r_1P_{11} & \cdot & -r_1P_{13} \\ \vdots & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ P_{61} & \cdots & P_{64} & 0 & 0 & 0 \\ P_{71} & \cdots & P_{74} & -r_7P_{71} & \cdot & -r_7P_{73} \end{bmatrix}^{-1} \cdot \begin{bmatrix} r_1P_{14} \\ \vdots \\ \cdot \\ \cdot \\ 0 \\ r_7P_{74} \end{bmatrix} \quad (6a)$$

$$\begin{bmatrix} G_1 \\ \vdots \\ G_4 \\ _bH_1 \\ \cdot \\ _bH_3 \end{bmatrix} = \begin{bmatrix} P_{11} & \cdots & P_{14} & -x_1P_{11} & \cdot & -x_1P_{13} \\ \vdots & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ P_{61} & \cdots & P_{64} & 0 & 0 & 0 \\ P_{71} & \cdots & P_{74} & -x_7P_{71} & \cdot & -x_7P_{73} \end{bmatrix}^{-1} \cdot \begin{bmatrix} x_1P_{14} \\ \vdots \\ \cdot \\ \cdot \\ 0 \\ x_7P_{74} \end{bmatrix}. \quad (6b)$$

θ_7 is iterated, and the constants re-evaluated until the difference between the $\{_aH_i\}$ and $\{_bH_i\}$ has been minimized. The measure D of this difference is taken as

$$D(\%) = \frac{100}{3} \sum_{i=1}^{3} \frac{|_aH_i - {_bH_i}|}{|_aH_i + {_bH_i}|/2}. \quad (7)$$

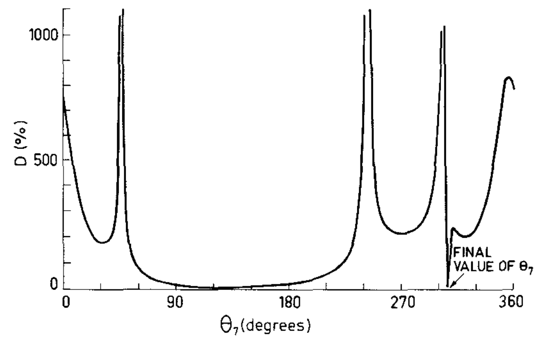

Fig. 2. Plot of a difference measure D between $\{_aH_i\}$ and $\{_bH_i\}$ versus the assumed angle of the intermediate calibrating reflection standard, computed from the data of a calibration at 5 GHz. It is seen that the correct angle is distinguished by the sharpness of the minimum in its neighborhood.

The six-port constants $\{H_i\}$ are taken to be the means of the corresponding members of the final $\{_aH_i\}, \{_bH_i\}$.

Ideally, it should be possible to iterate θ_7 until the $\{_aH_i\}$ and $\{_bH_i\}$ are identical. That this is not so in practice is an indication of inaccuracies in the descriptions of the calibrating standards, of errors in the measurement of the $\{P_{ni}\}$, and of computational limitations. The minimized value $D_{\min}$ is a measure of these inaccuracies and errors, and it being unusually large (>5 percent) has proven useful on several occasions to draw early attention to dirty or loose connectors, ill-chosen phasing of the calibrating short-circuits [7], or the supply of inaccurate data by using, for example, an inferior-quality sliding short-circuit for calibration purposes.

Fig. 2 shows the variation of D with θ_7 for a six-port being calibrated at 5 GHz, and in particular shows the distinctive rapid variation of D near the correct value of θ_7. If a simple minimum-seeking routine is to locate the final minimum, the initial estimate of θ_7 must be within about 5° of the final value. This initial estimate is obtainable from either the visual display, or a computed curve such as Fig. 2.

IV. Comparison of the Two Calibration Methods

Method B requires one more phased short-circuit standard for the six-port calibration than Method A. However, once the iteration of the phase of the intermediate reflection coefficient has been terminated, the additional redundancies of Method B provide a measure ($D_{\min}$) of the calibration accuracy, whereas Method A, having used eleven equations to determine eleven constants, provides no such measure. It has been our experience that the need to provide the extra phased short-circuit for Method B is more than compensated by the confidence engendered when $D_{\min}$ is found to have a low value.

Both methods involve the solution of matrix equations at each iteration. Because only one line of the matrices is affected with each iteration, successive solutions may be obtained rapidly using the algorithm in Appendix B.

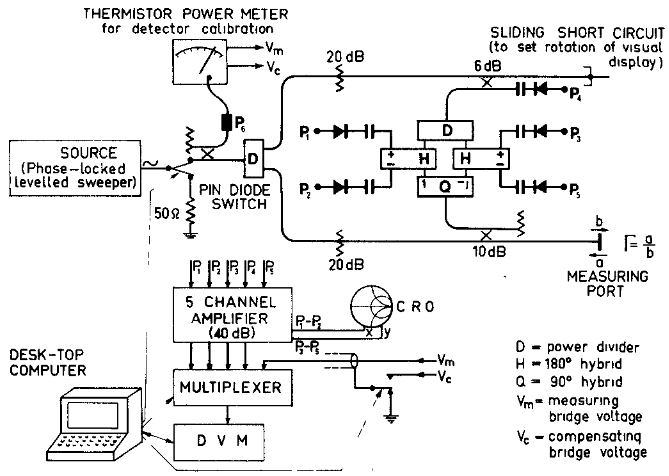

Fig. 3. Basic elements of a six-port and of the associated circuitry. An approximate, visual display of Γ is obtained by simple analog means in real time, and a precise value for Γ is obtained after six-port calibration and computation.

V. Six-Port Configuration

The circuit chosen as a six-port reflectometer from 1–8 GHz, and to provide a visual display of the unknown reflection coefficient, is shown in Fig. 3. The measuring port, and the calibrating standards are equipped with GR 900-type connectors. Either a beadless sliding short-circuit or a set of GR 900 LZ beadless precision airlines with a terminating short-circuit is used for calibration purposes, and a similar choice is available for the calibrating matched termination. The intermediate termination is either a wide-band mismatch (GR 900-W200), the VSWR of which was measured by independent means [11], or a reflection standard comprising a sliding load inside a precision coaxial line of different characteristic impedance.

If b is the signal incident on the unknown termination, and a is the signal reflected from it, then $\Gamma = a/b$ and the powers P_1, P_2, P_3, P_4, P_5 are approximately proportional to $|\alpha b + a|^2$, $|\alpha b - a|^2$, $|\alpha b - ja|^2$, $|b|^2$, $|\alpha b + ja|^2$, respectively, where $\alpha \simeq 1.6$ is the ratio of the couplings of the 6-dB and 10-dB couplers.

A visual display of Γ is available if the differences $P_1 - P_2$ and $P_3 - P_5$ are formed electronically and applied to the X and Y deflection inputs of the CRO or X–Y recorder. The display may be rotated by adjusting the position of the short-circuit attached to the 6-dB coupler. The circuit arrangement for the direct display is identical to that of the locating reflectometer when used in the Smith Chart mode [10]. The uncertainty associated with the display depends upon the quality of the circuit components, including parameters such as coupler imperfections, and power meter nonlinearity. Typically, uncertainties of ±5 percent can be expected.

Hoer has shown [5] that the outputs P_1, P_2, P_3, P_5 form a class unsuited for use with six-port theory. For this reason P_4 is used with any three of these four in the application of (1). P_6 is a reference power meter used only during the calibration of the four diode detectors selected for the six-port measurement.

From an inspection of the circuit in Fig. 3 or from an examination of the location of the centers of the Engen circles [6], the dynamic range demanded of the power meters in order to cope with any passive termination, is found to be 13 dB. In practice, because of losses and component tolerances, a 16-dB dynamic range is required.

The recording of power meter readings is automated using a desk-top computer to control a multiplexer, DVM, and RF switch. Because of thermal EMF's and amplifier offsets, every RF measurement is preceded by measuring the individual channel's dc level with the RF off, and subtracting this from the channel voltage when the RF is on. The DVM is programmed to take a set of 25 readings of each voltage. If, after each set, the standard deviation of the mean is greater than a preselected value, an additional set of 25 readings is taken [12]. If, after 400 readings, the preselected threshold is not achieved, these readings are rejected and the process is restarted. This technique has proven useful for detecting instabilities such as signal transients, loose RF connectors, and flexing cables.

VI. Power Meters

Six-port theory requires that the input impedance of each power meter remain constant over the range of power incident upon it. The input impedances need not be known, nor need they be the same. Any signal harmonics capable of detection should be filtered before reaching the power meters.

To avoid the high-power sources needed for six-port devices using thermal power meters, diode detectors are used as power meters. In a coaxial system it is necessary to insert dc blocking capacitors between the power meter ports and the detectors to ensure that the rectified currents do not interact with other detectors.

The detectors chosen were Hewlett-Packard low-barrier Schottky diodes (type HP 33330B) which, according to the manufacturer's specifications, exhibit near-zero temperature coefficients with video loads of about 3.3 kΩ. The detector amplifier channels were provided with these input resistances, and variations in ambient temperature of $\pm 1°C$ were found to affect the magnitude of a measured reflection coefficient by less than 0.0005.

The maximum power incident upon any of the detectors is restricted to about 250 μW. Tests using a range of powers of this or less, indicated that the diode reflection coefficient variation was less than 0.002.

Because the output voltages of semiconductor diodes are not linear with input power, the diodes must be calibrated initially. If the power P_i incident on the ith detector is related to the output voltage V_i by $P_i = k_i f_i(V_i)$, where k_i is a constant, then (1) may be written as

$$\Gamma = \sum_{i=1}^{4} (F_i' + jG_i') f_i(V_i) \Big/ \sum_{i=1}^{4} H_i' f_i(V_i) \qquad (8)$$

where the proportionality constants have been absorbed into the six-port constants. Therefore it is not necessary to know the $\{k_i\}$, but only the relationship between microwave power at any point in the system, and the (amplified) output voltage of the relevant detector. This property allows the detectors to be calibrated *in situ*—they need never be removed from their respective ports.

The detector calibration proceeds after a particular detector channel has been selected by the multiplexer,and the residual dc level recorded with the RF turned off. Then, with the RF turned on, a sliding short-circuit connected to the measuring port is positioned until the power into the selected detector is maximized. Both the dc output of the channel and the RF power as measured by power meter P_6 in Fig. 3 are recorded for 10–20 power level settings covering the 16-dB dynamic range over which the detector is to be used. These level settings can be automated using a programmable step attenuator.

Power meter P_6 is a thermistor type which exhibited linearity better than 0.001 dB when checked against a standard piston attenuator. Because only the relative powers at the four detector ports are important in (1), the absolute accuracy of the power meter P_6 need not be known.

It is convenient to fit the logarithms of RF power as measured by power meter P_6, and dc voltage from the detector being calibrated, with a 5th degree polynomial in the least squares error sense. The standard deviation of this fit is typically 0.001 dB. For highest accuracy, the detectors are calibrated at the frequency of use, and the polynomial coefficients for each calibration frequency are stored in the computer, to be recovered when needed.

VII. Adaption to Other Transmission Systems

The six-port would be restricted in its application if it were limited to measurements in the GR 900 connector system. The use of transformers to adapt to other transmission line systems (including waveguide), as shown in Fig. 4, seemingly requires a set of calibrating standards for each new system.

Instead, the six-port computer has been programmed to calculate the S parameters of a transformer from the GR 900 connector to a second system, and to use these parameters and the six-port calibration constants to compute reflection coefficients referenced to a chosen plane in the second system. As shown below, this requires only a short-circuit, an offset short-circuit, and a matched termination in the second system. As a by-product, the S parameters of the transformer are available.

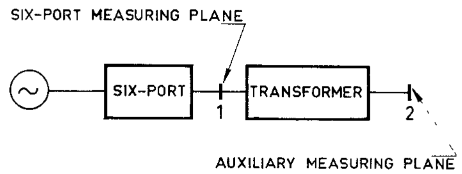

Fig. 4. Extension of the six-port measuring plane through a calibrated transformer.

If a termination with reflection coefficient Γ_2 is connected to auxiliary reference plane 2 in Fig. 4, the reflection coefficient measured at the six-port reference plane 1 is given by (1) and by

$$\Gamma = S_{11} + \frac{S_{21}^2 \Gamma_2}{1 - S_{22}\Gamma_2} \qquad (9)$$

where the S-parameters are those of the transformer. These parameters are calculable from the three values of Γ measured when two phased short-circuits and a matched termination are connected successively to the auxiliary reference plane.

The reflection coefficient Γ_x of an unknown termination connected to the auxiliary plane 2 may then be computed using

$$\Gamma_x = \frac{\Gamma - S_{11}}{S_{21}^2 + S_{22}(\Gamma - S_{11})} \qquad (10)$$

where Γ is given by (1).

VIII. Measurement Accuracy

One test of the accuracy of the six-port was performed at 3 GHz after connecting a precision sliding short-circuit to the measuring port. The short-circuit was moved in equal increments over a half-wavelength and the reflection coefficient calculated. The measured phase shift tracked the nominal phase shift within 0.1°, and the standard deviation of the magnitude of the measured reflection coefficients was less than 0.0006.

The ability of a six-port that has been calibrated at plane 1 in Fig. 4 to measure a reflection coefficient referenced to plane 2 permits some self-checking of the accuracy of the system. If the reflection coefficient of a termination is measured directly at plane 1, and then remeasured when connected to plane 2 of the transformer, the two reflections seen by the six-port at plane 1 will be, in general, quite different. A measure of the accuracy of the six-port and of the representation of the transformer can be obtained by comparing the two values calculated for the reflection coefficient of the termination.

One transformer chosen for this test was a well-matched attenuator whose S parameters were determined using the method outlined in Section VII. The magnitude of S_{21} was measured as 6.015 dB, which may be compared with its accurately known value of 6.017 dB. A reflecting termination was connected directly at plane 1 and then at plane 2. The magnitude of the reflection coefficients calculated from these two measurements agreed to within 0.0003, despite the 12-dB path loss introduced by the attenuating transformer.

IX. Conclusion

A low-powered low-cost six-port reflectometer has been described and shown to be capable of high precision using diode detectors as power meters. The detectors are calibrated *in situ*, and need never be removed from their respective ports. An extra (fifth) diode detector enables the measurand to be displayed in real time with moderate accuracy, using minimal analogue circuitry.

The six-port is calibrated using a matched termination, an intermediate reflection (preferably $0.3 \leq |\Gamma| \leq 0.7$), and either four or five phased short-circuits. The phase of the intermediate reflection need not be known *a priori*.

Measurements may be made in an alternative transmission line system by the provision of transformer, two phased short-circuits, and a matched termination in that system.

Appendix A

Associated with a sliding short-circuit (SSC) are losses due to both finite conductivity, and imperfect contact between the sliding component and the conductors. The losses present when the SSC is positioned nearest to its port may be termed fixed losses, and as the short-circuit is moved further from the port the additional losses which are incurred are calculable from line length and conductivity.

During six-port calibration, these additional losses can be allowed for by providing a value of the short-circuit reflection coefficient dependent on the position of the SSC. Alternatively, the resolution and the stability of the six-port permit the determination of the effective velocity of propagation, and the additional loss of the SSC by measurement. Measurements are taken with the SSC in two positions exactly half-wavelength apart as indicated by the six-port. The difference in the micrometer readings is the effective half-wavelength (including the effects of actual velocity of propagation and possible screw-pitch error), and from the change in $|\Gamma|$ the line loss over a half-wavelength is calculable.

A measure of the fixed loss can be determined after the calibration by connecting an (ideally lossless) short-circuit in the plane of the measuring port. The apparent reflection coefficient Γ_a will have a magnitude greater than unity. It is then possible to recalculate the six-port constants taking the SSC reflection coefficients to be $1/|\Gamma_a|$ times their values used previously. Alternatively, and more efficiently, the calibrating intermediate reflection coefficient can be scaled by multiplying by $|\Gamma_a|$, and the six-port constants recomputed using the algorithm of Appendix B. All subsequent reflection coefficients measured using the six-port need then be multiplied by $1/|\Gamma_a|$.

The reference reflection of unit magnitude is then the single planar short-circuit connected to the six-port for normalizing, and is also convenient to use as a well-defined phase reference plane.

Appendix B

Assume that the matrix equation of order $n+1$

$$\begin{bmatrix} \boldsymbol{y}_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}_1 & d_1 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{x} \\ w_1 \end{bmatrix} \tag{B1}$$

has been solved for the $\boldsymbol{y}, z_1$, where $\boldsymbol{y}_1, \boldsymbol{b}, \boldsymbol{x}$, are column vectors and $\boldsymbol{c}_1$ a row vector each of length n, $\boldsymbol{A}$ is an $n \times n$ matrix, and z_1, d_1, w_1 are scalar. If the last row in the matrix equation is now changed, we wish to find the solution $\boldsymbol{y}_2, z_2$ to

$$\begin{bmatrix} \boldsymbol{y}_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}_2 & d_2 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{x} \\ w_2 \end{bmatrix}. \tag{B2}$$

Equations (B1) and (B2) may be rewritten as

$$\begin{bmatrix} \boldsymbol{x} \\ w_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}\boldsymbol{y}_i + \boldsymbol{b}z_i \\ \boldsymbol{c}_i\boldsymbol{y}_i + d_i z_i \end{bmatrix}, \qquad i = 1, 2. \tag{B3}$$

Subtracting these two equations gives

$$\begin{bmatrix} \boldsymbol{0} \\ w_2 - w_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}(\boldsymbol{y}_2 - \boldsymbol{y}_1) + (z_2 - z_1)\boldsymbol{b} \\ \boldsymbol{c}_2\boldsymbol{y}_2 + d_2 z_2 - \boldsymbol{c}_1\boldsymbol{y}_1 - d_1 z_1 \end{bmatrix} \tag{B4}$$

and hence

$$z_2 = z_1 + \frac{(w_2 - w_1) + (\boldsymbol{c}_1 - \boldsymbol{c}_2)\boldsymbol{y}_1 + (d_1 - d_2)z_1}{d_2 - \boldsymbol{c}_2\boldsymbol{A}^{-1}\boldsymbol{b}} \tag{B5}$$

$$\boldsymbol{y}_2 = \boldsymbol{y}_1 + (z_1 - z_2)\boldsymbol{A}^{-1}\boldsymbol{b}. \tag{B6}$$

Notice that (B5) and (B6) involve the term $\boldsymbol{A}^{-1}\boldsymbol{b}$, so $\boldsymbol{A}$ must be inverted. However this only needs to be done once, and the same $\boldsymbol{A}^{-1}\boldsymbol{b}$ can then be used repeatedly to find solutions of any matrix equation of the form of (B2), where only one line of the original matrix equation has been changed.

Acknowledgment

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Short Papers

A Broad-Band Amplifier Output Network Design

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Abstract—**An analytic design method for a lossy gain-compensating network is presented and the advantages of lossy networks are discussed. Examples of two-stage amplifiers using FET's and bipolar transistors are presented to show the feasibility of this particular network in low power amplifier designs. These amplifiers obtain gains of 15.4±0.5 dB with a 2.5-dB maximum noise figure in the 4.0–6.0-GHz frequency range and 16.5±1.2 dB with a maximum input VSWR of 1.78:1 over the 1.0–2.0-GHz frequency range, respectively.**

I. Introduction

Lossy gain-compensating output networks can provide lower input reflection coefficients, a lower amplifier noise figure, and a more predictable amplifier design [1]. The resistive nature of this type of network may also improve amplifier stability and distortion by reducing standing waves within the amplifier. Although lossy broad-band gain-compensating networks are often used [1]–[4], explicit, analytic design techniques for these networks have not been reported.

This paper presents an output circuit design based upon a π matching network combined with a bandpass/bandstop diplexer. As a result, this network contains both the drain supply inductance and the dc blocking capacitor, which are needed in any output network, as integral elements. Explicit formulas for the element values of this network are derived and presented shortly. This method is different from previous methods [1] because this technique allows the device load to approach 50 Ω as frequency is decreased. However, this network is similar to that used in [2] and [3] in that the supply voltage is inserted through a quarter-wavelength shunt stub and a series resistor. This method results in greater stability and tunability for the amplifier.

A bandpass/bandstop diplexer is more useful than a simple low-pass/high-pass diplexer because it provides an exact match at one frequency, and an arbitrary amount of attenuation (limited only by network element Q's) at any frequency. Diplexing networks may be used in either input or output networks depending

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